

# Modeling of Surfzone Bubbles Using a Multiphase VOF Model

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## 1. INTRODUCTION

The ability to make optically-based observations in the surf-zone is strongly influenced by the presence of suspended sediment particles and of air bubbles, both of which are present due to the action of breaking waves. Wave breaking is instrumental in injecting large volumes of air into the water column. This air volume subsequently evolves into a distribution of bubble sizes that interact with the fluid turbulence and are advected by the organized flow. The bubble population in the surf-zone is intensified due to the greater intensity of breaking processes, leading to increase in turbulence intensity and associated energy dissipation. The bubble sizes are also affected by the densely sedimented flows that could alter the relationship between turbulent perturbing forces and surface tension-based restoring forces leading to the determination of critical bubble diameters.

Models for the distribution of bubble populations in the surfzone are rare, and, where they exist, are based on a simplified view of the circulation process of interest without involving detailed processes of bubble injection, interaction and evolution (for example, Vagle et al., 2005). The problem of bubble injection and initial distribution in the water column is happening on the time scale of the individual waves. A prediction of the distribution of bubbles over depth due to a breaking event depends on a good representation of the fluid velocity field at the wave-resolving time scale. In the present study, we formulate a general, multiphase model representing water, multi-component sediment and multi-component bubble populations. The multiphase model incorporates not only the interaction between fluid and sediment phase and fluid and bubble phase, but between sediment and bubble phases as well.

## 2. FORMULATIONS

The three-phase model can be obtained by ensemble averaging the conservation equations for each phase in a multiphase flow, following Buscaglia et al. (2002).

### 2.1 The mixture fluid phase:

$$\nabla \cdot \mathbf{u}_m = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial \mathbf{u}_m}{\partial t} + \mathbf{u}_m \cdot \nabla \mathbf{u}_m + \frac{1}{\rho_0} \nabla P_m \\ = \frac{1}{\rho_0} \nabla \cdot [\mu_T (\nabla \mathbf{u}_m + \nabla^T \mathbf{u}_m)] - \frac{\rho_m}{\rho_0} g \mathbf{k} \end{aligned} \quad (2)$$

where  $\mathbf{u}_m$ ,  $P_m$  and  $\rho_m$  represent the mixture quantities of fluid velocity, pressure and density, respectively.  $\rho_0$  is the so called reference density which has replaced  $\rho_m$  in all terms but the gravity term using the Boussinesq approximation.  $\mathbf{k}$  is a vertical unit vector.  $\mu_T$  is the eddy viscosity coefficient which is related to  $k$  and  $\epsilon$  in the  $k - \epsilon$  turbulence equations:

$$\mu_T = \rho_0 C_\mu \frac{k^2}{\epsilon} \quad (3)$$

where  $C_\mu = 0.09$

The buoyancy force (the last term in (2)) can be evaluated as

$$\frac{\rho_m}{\rho_0} g \mathbf{k} = [1 - \alpha_b + \alpha_s (\rho_s / \rho_w - 1)] g \mathbf{k} \quad (4)$$

where  $\alpha_b$  and  $\alpha_s$  are the volume fractions of bubbles and sediment following the definitions in Drew and Passman (1998).  $\rho_w$  and  $\rho_s$  represent wave density and sediment density, respectively.

### 2.2 Bubble phase:

$$\frac{\partial C_b(i)}{\partial t} + \nabla \cdot (C_b(i) \mathbf{u}_g) = \mathcal{S}_c + \nabla \cdot (\mathcal{D}_g \nabla C_b(i)) \quad (5)$$

$$\frac{\partial N_b(i)}{\partial t} + \nabla \cdot (N_b(i) \mathbf{u}_g) = \mathcal{S}_n + \nabla \cdot (\mathcal{D}_g \nabla N_b(i)) \quad (6)$$

where  $C_b(i)$  and  $N_b(i)$  represent, respectively, the gas molar concentration and bubble number per unit volume for bubble size  $i$ .  $\mathbf{u}_g$  is the bubble advection velocity which can be calculated by

$$\mathbf{u}_g = \mathbf{u}_m + w_s(r_b) \mathbf{k} \quad (7)$$

in which  $w_s(r_b)$  is the bubble-slip velocity, assumed only depending on the bubble radius.  $S_c$  and  $S_n$  are source/sink terms associated with inter-phase adjustment of bubble quantity between different component  $i$  caused by bubble size changes due to pressure change, bubble-sediment interaction, bubble breakup and coalescence.

$\mathcal{D}_g$  is the dispersion coefficient associated with the turbulence and bubble-sediment interaction. In the isotropic model proposed by Carrica et al. (1998),

$$\mathcal{D}_g = \frac{\mu_T}{\rho_0 S_g} \quad (8)$$

in which  $S_g$  is the Schmidt number for gas.

The gas volume fractions used in (4) can be calculated using

$$\alpha_g = \frac{\mathcal{R}T_g \sum C_b(i)}{P_g} \quad (9)$$

where  $\mathcal{R}$  is the universal gas constant, i.e., 8.314 J/mol K.  $T_g$  is the absolute gas temperature.  $P_g$  is gas pressure, assumed equivalent to  $P_m$ .

As a simple approach, the source/sink terms in (5) and (6) can be evaluated only based on bubble size changes due to pressure without take into account sediment effect, bubble breakup and coalescence. Bubble radius can be calculated using

$$r_b(i) = \left( \frac{3\nu_b(i)}{4\pi} \right)^{1/3} \quad (10)$$

where  $\nu_b(i)$  is the bubble volume of component  $i$  which can be obtained by

$$\nu_b(i) = \frac{C_b(i)\mathcal{R}T_g}{P_g N_b} \quad (11)$$

### 2.3 Sediment phase:

$$\frac{\partial C_s(i)}{\partial t} + \nabla \cdot (C_s(i)\mathbf{u}_s) = \nabla \cdot (\mathcal{D}_s \nabla C_s(i)) \quad (12)$$

where  $C_s(i)$  is the sediment concentration with a grain size numbered as  $i$ .  $\mathcal{D}_s$  is the coefficient for sediment dispersion which links to turbulence and bubble-sediment interaction.  $\mathbf{u}_s$  is the sediment advection velocity which can be evaluated by

$$\mathbf{u}_s = \mathbf{u}_m - w_s(r_s)\mathbf{k} \quad (13)$$

in which  $w_s(r_s)$  is the sediment particle fall velocity with respect to grain size  $r_s$ . Pick-up function is used at the bottom boundary.

### 3. MODEL IMPLEMENTATION AND APPLICATIONS

We used the 2-DV VOF model, RIPPLE, as a basic fluid component. The VOF model has been enhanced with several different turbulence closure models such as  $k-\epsilon$  model and multi-scale LES model. The governing equations (5) and (6) for the bubble phase and (12) for the sediment phase have been implemented based on the existing Lagrangian tracer subroutines. The connection between the intensity and size spectra of entrained bubbles and turbulence levels in the surfzone is formulated following the previous studies by Terrill et al., 2001, Garrett et al., 2000, etc.

Applications of the present model include an idealized case indicating bubble evolution and buoyancy induced circulations, a case showing bubble and sediment interaction, and modeling of bubble generation and transport on a sandy beach. Some preliminary results can be seen at <http://coastal.udel.edu/~fyshi/bubbles.html>. Detailed model results and analysis will be presented in the conference.

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