OBJECTIVE
The purpose of the experiment is to demonstrate typical flow rate measurement devices for incompressible fluids in pipes: (1) Venturi meter, (2) Orifice plate, and (3) Rotameter. Each flow measurement device will be compared to the standard method of using the catch-tank and stopwatch to measure the flow rate. In addition, you are also asked to determine the energy loss incurred by each of these devices. We will use extensively the Bernoulli equation and steady-state energy equation.

APPARATUS
Figures 1 and 2 show the details of the apparatus for this experiment. Water from the hydraulic bench enters the equipment through a Venturi meter consisting of a short nozzle, a throat, and a long diffuser (see Figure 2, A-B). Next, the water flows into a rapidly diverging section (C-D), followed by a settling length where the flow again adjusts to uniform conditions, and then through an orifice plate (E-F). Following a further settling length and a right-angle bend, the flow enters a rotameter (H-I). The rotameter is a transparent tube of gradually diverging cross-section in which the “float” takes an equilibrium position; the vertical position of the float is a measure of the flow rate.

Figure 1: The apparatus of this experiment.
After the rotameter, the flow returns via a control valve to the hydraulic bench where the flow rate can be measured using the catch-tank and a stopwatch. This Q is termed $Q_m$. The test-section has nine pressure taps (A to I) as shown in figure 2, each of which is connected to its own manometer for immediate read-out.

![Figure 2: Explanatory diagram of the flow measurement apparatus](image)

**THEORY**

**Flow Measurement**

1. **Venturi Meter**

   In the converging section of the Venturi meter, the flow is accelerated continuously, and therefore, the losses are considered small. It is appropriate to apply the Bernoulli equation between A and B. You are asked to show that the volume flux $Q_v$ is given by

   \[
   Q_v = A_B \sqrt{\frac{2g(h_A - h_B)}{1 - (A_B/A_A)^2}}
   \]

   where $A_A$ and $A_B$ are the cross-sectional areas, and $h_A$ and $h_B$ are the measured heights in the manometer tubes at A and B, respectively.

2. **Orifice Meter**
The orifice meter is a plate with a central hole introduced into the flow path. It is the easiest and cheapest to install the plate between existing pipe flanges. However, the energy loss associated with the orifice meter is large. The mechanical energy equation between E and F gives:

\[
\frac{P_E}{\gamma} + \frac{V_E^2}{2g} = \frac{P_F}{\gamma} + \frac{V_F^2}{2g} + h_{LEF}
\]

(2)

where \( \gamma \) is the specific gravity of water and \( h_{LEF} \) is the corresponding head loss. Typically, the energy loss is expressed in terms of the piezometric head loss using a coefficient \( C_o \), so that:

\[
P_E - P_F - h_{LEF}\gamma = C_o^2(P_E - P_F)
\]

(3)

Show that the volume flux \( Q_o \) can be written as

\[
Q_o = A_F C_o \sqrt{2g(h_E - h_F)} \over \sqrt{1 - (A_F/A_E)^2}
\]

(4)

Here, \( A_E \) is the tube area upstream of the orifice plate, and \( A_F \) is the flow area at section F. Due to flow separation \( A_F \) can be taken to be the area of the orifice opening. For the orifice plate given in this experiment, \( C_o = 0.6 \).

3. Rotameter
The “float” inside our rotameter is actually made of steel. The reason it “floats” up when the flow is turned on is due to the drag force exerted by the water as it flows in the annular gap between the float and the rotameter tube. The rotameter tube has a diverging cross-sectional area. (Explain why this is needed.) As a consequence, the float has one unique position for each flow rate. The higher the flow rate, the higher the float position. Determine the float position which will be used to estimate the flow rate.

Energy Losses

1. Venturi Meter
While the nozzle section has small losses, it is not true for the diverging section of the Venturi meter. Apply the mechanical energy equation between sections A and C and show that

\[
h_{LAC} = h_A - h_C
\]

(5)

Express the energy loss as a fraction of the inlet velocity head \( V_A^2/2g \) at section A. (You can determine \( V_A \) by knowing the flow rate \( Q_m \) and the area \( A_A \).)

2. Orifice Meter
The energy dissipation due to the orifice meter can be obtained by using the cross sections at E and G where the flow may be assumed to be uniform over the cross section (section G is chosen so that it is already downstream of the flow separation regime):

$$h_{LEF} = h_E - h_G$$

(6)

On the other hand, equation (3) can be written as:

$$h_{LEF} = (1 - C_o^2)(h_E - h_F)$$

(7)

Determine your measured value of the piezometric head differences between E-F and E-G and compare with the head difference ratio based on equations (6) and (7). Express the energy loss due to the orifice meter in terms of the inlet velocity head at section E (by taking a ratio of the two quantities). It shall be sizable.

3. Rotameter

Apply the mechanical energy equation between H and I. You may ignore the velocity head change between H and I, because the rotameter tube diverges rather gradually (areas are approximately same, so velocities are approximately equal). Show that:

$$h_{LHI} = h_H - h_I$$

(8)

The energy loss for the rotameter is seen to be approximately constant irrespective of Q. Express the energy loss in terms of the inlet velocity head at section G. Show that for small Q the energy loss will be many times the inlet velocity head.

PROCEDURE

For 10 different flow rates (start at 2 cm and work your way up in ~2 cm increments on the rotameter) note down the following:

a. The manometric heights $h_A, h_B, \ldots, h_I$.
b. The rotameter reading in centimeters.
c. The volume of water collected in the catch tank $V$ in time $t$ to find the flow rate $Q_m$.

DATA ANALYSIS AND DISCUSSION

a. Derive equations (1), (4), (5), (6), (7) and (8).

b. Prepare a calibration graph for the rotameter: $Q_m$ (on y-axis) vs. rotameter reading in cm (on x-axis). Determine the calibration constants by a least-square straight line fit to the data and thereby determine an expression for $Q_r$ obtained from the rotameter reading.
c. Determine the volume flow rate $Q$ as given by
   i. the Venturi meter $Q_v$,
   ii. the orifice plate $Q_0$, and
   iii. the Rotameter $Q_r$ (using the calibration constants you just determined).

d. Compare these flow rates with the most direct method using the catch-tank and stopwatch. Plot the volume flow rate from all four devices on the $y$-axis of the same graph against the serial number of the reading on the $x$-axis. The differences can be highlighted if you plot $(Q_{\text{meter}} - Q_m)$ for each device on a separate graph where $Q_{\text{meter}} = Q_v, Q_0, \text{and } Q_r$.

e. Plot the energy loss due to the various devices as a function of $Q_m$ in two ways: (i) plot the energy loss in mm; and (ii) as a fraction of the velocity head at the inlet of each device. How does the energy loss vary with Reynolds number at the inlet of each device? Discuss your results. Based on your results, which device would you recommend for flow measurement? Discuss the criteria you use for your choice. Note that each device has its own advantages and disadvantages.

f. Explain why the rotameter must have a slightly diverging cross-section?